

# Introduction to Structural Equation Modeling

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# Outlines

- Introduction
  - SEM
  - Terms in SEM
  - Path diagram
- Relationships in SEM
  - Correlation
  - Causal
  - Mediation
  - Moderation

# Expected outcomes

- Understand and apply the basic knowledge of SEM
- Specify SEM models involving correlation, causal, mediation & moderation, and interpret the results

# Introduction

# SEM

- Structural Equation Modeling (SEM) is a multivariate statistical modeling that aims to explain the structure of relationships among multiple variables (Hair, Black, Babin, and Anderson, 2010)
- Needs strong theoretical specification of the model ahead of the analysis → to verify our theory on the relationships

# SEM

- Consists of TWO components (Bartholomew et al., 2008):
  1. measurement model (CFA): dealing with latent variables (factors) and the relationships between the items and the factors
  2. structural model (path analysis): dealing with how latent variables are related to each other

# SEM

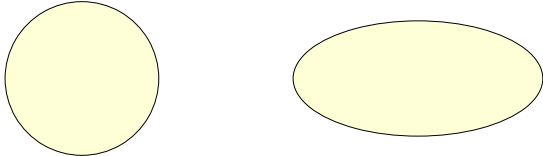
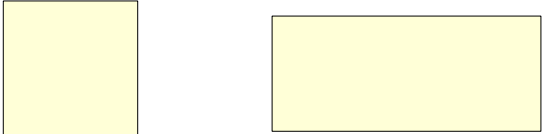


- Similar to CFA, how the variance-covariance matrix produced from the model fits the variance-covariance matrix of the observed data → Goodness of fit of model to the data

# Terms in SEM

| Term                       | Description                                                                                            |
|----------------------------|--------------------------------------------------------------------------------------------------------|
| <i>Exogenous variable</i>  | <u>Independent</u> , predictor variable. Could be observed (manifest) or unobserved (latent) variables |
| <i>Endogenous variable</i> | <u>Dependent</u> , outcome variable. Could be observed (manifest) or unobserved (latent) variables     |
| <i>Path diagram</i>        | A visual representation of the SEM model                                                               |



# Path Diagram

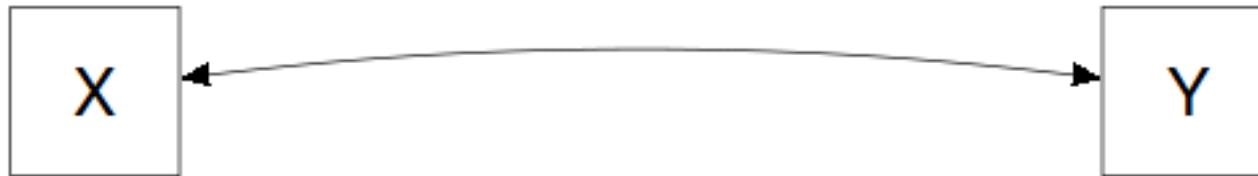
| <b>Concept</b>                        | <b>Diagram</b>                                                                        |
|---------------------------------------|---------------------------------------------------------------------------------------|
| <b><i>Latent variable</i></b>         |    |
| <b><i>Observed variable</i></b>       |    |
| <b><i>Correlation, covariance</i></b> |  |
| <b><i>Causal relationship</i></b>     |  |

# Relationships in SEM

# Correlation

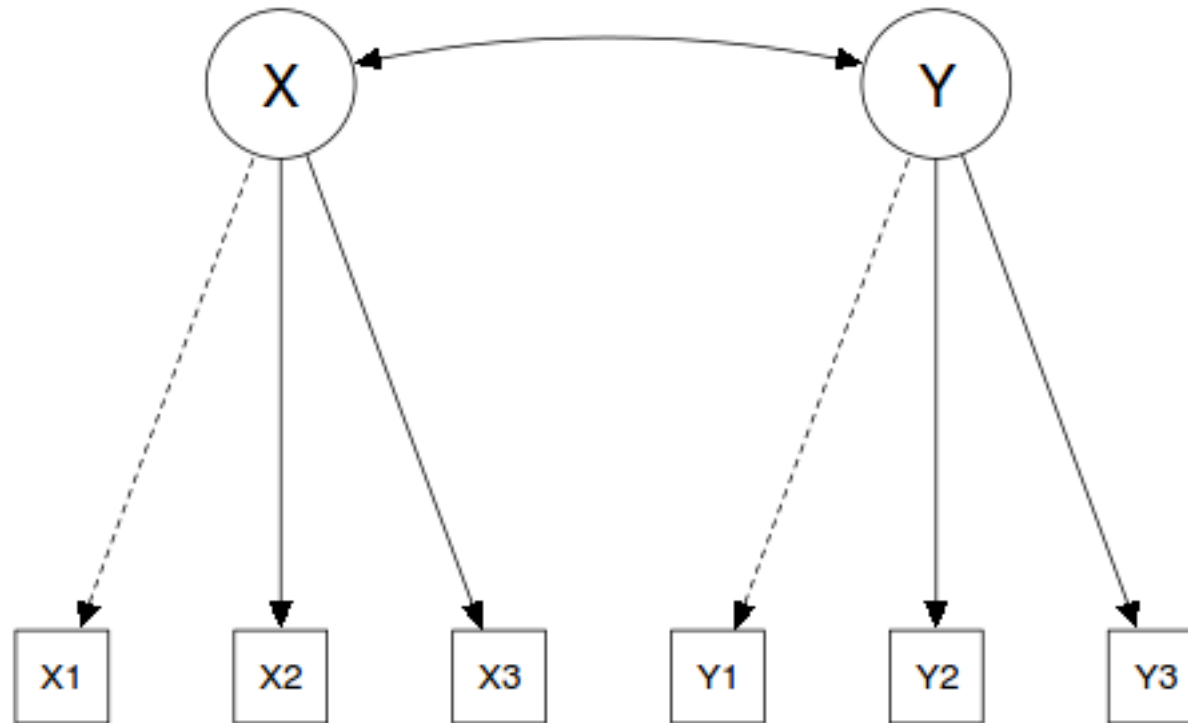
- Bidirectional correlation/covariance between variables (observed/latent)

# Correlation



Between observed variables

# Correlation

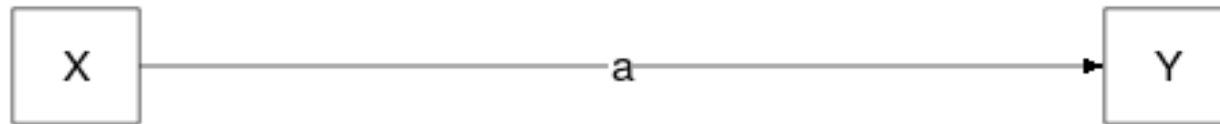


Between latent variables

# Causal

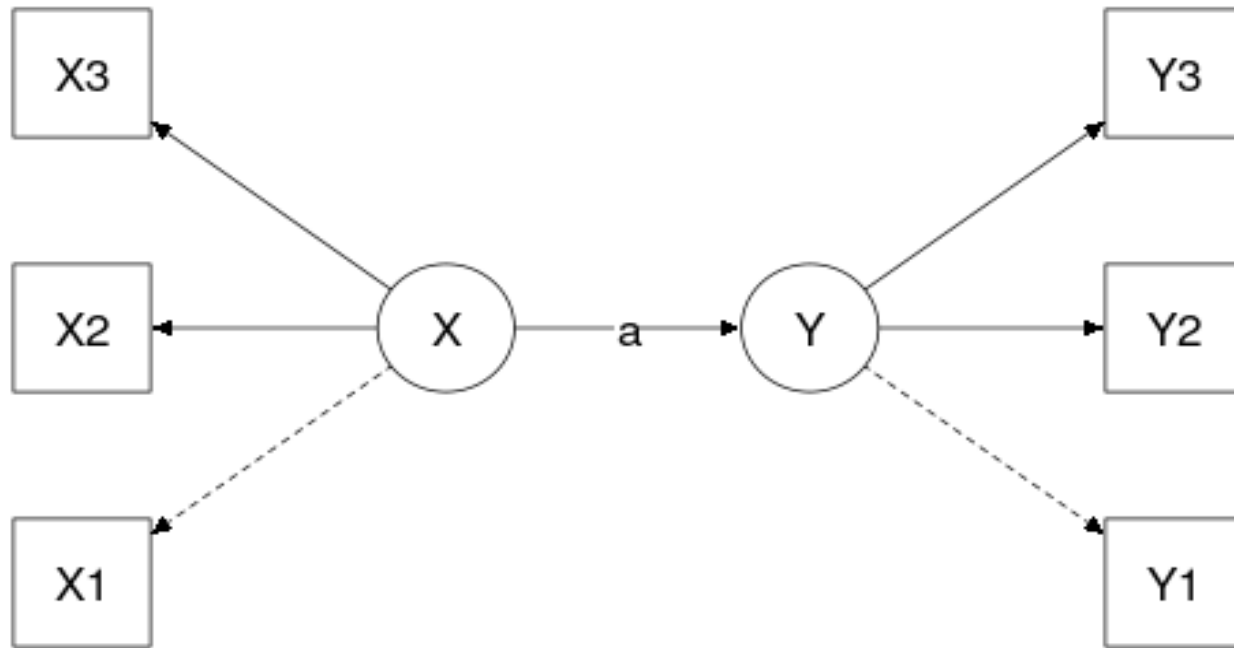
- Causal relationship between variables (observed/latent)
- Assign dependent and independent variables

# Causal



Endogenous/dependent Y caused by  
exogenous/independent X (observed)

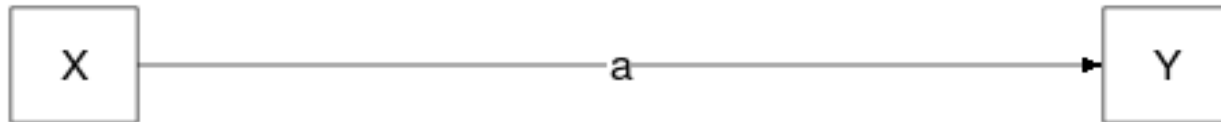
# Causal



Endogenous/dependent Y caused by  
exogenous/independent X (latent)

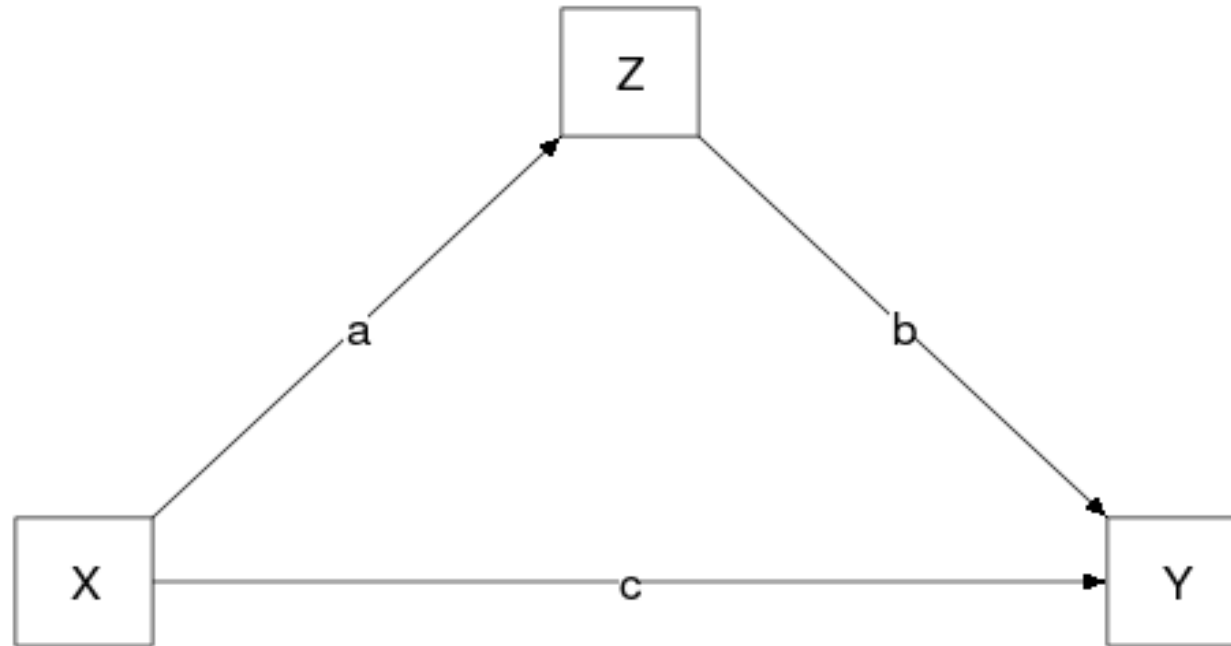


# Mediation



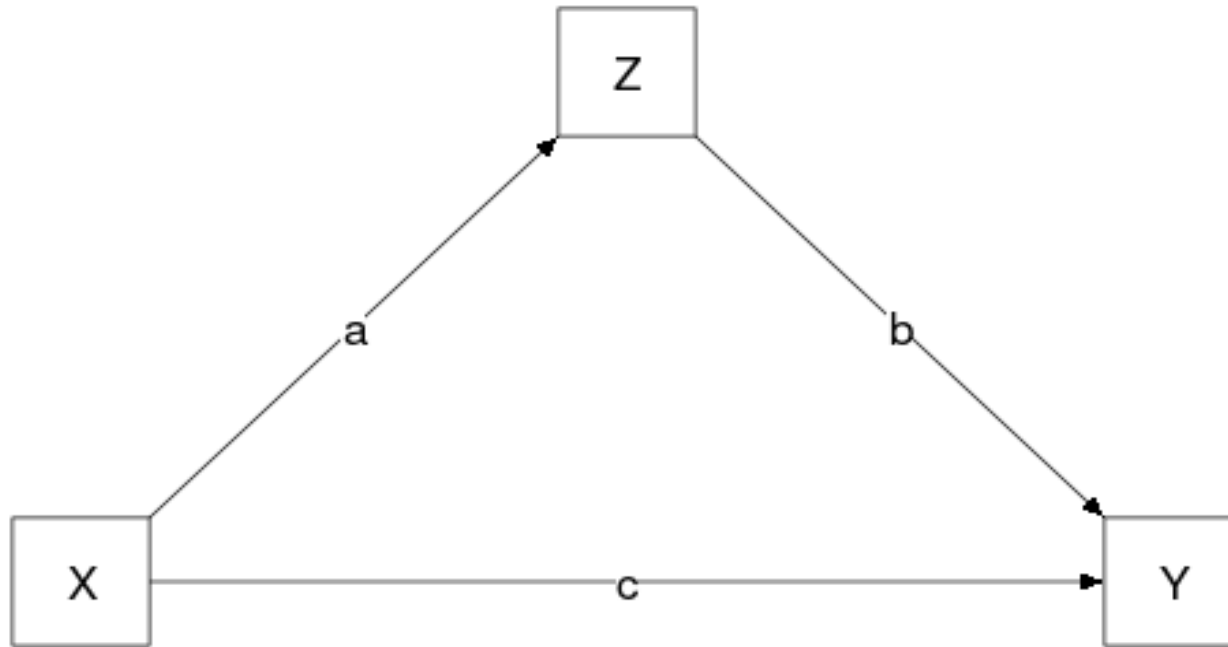
- Figure shows direct causal effect from independent X to dependent Y
- In SEM it is also possible to examine the effect of a mediating variable (mediator) Z on this established relationship

# Mediation



- A mediator  $Z$  is a third variable that intervenes between two related variables (Hair et al., 2010)
- Two roles, as an DV in one equation and IV in another (Awang, 2012)
- $Z$  mediates relationship of  $X$  to  $Y$

# Mediation

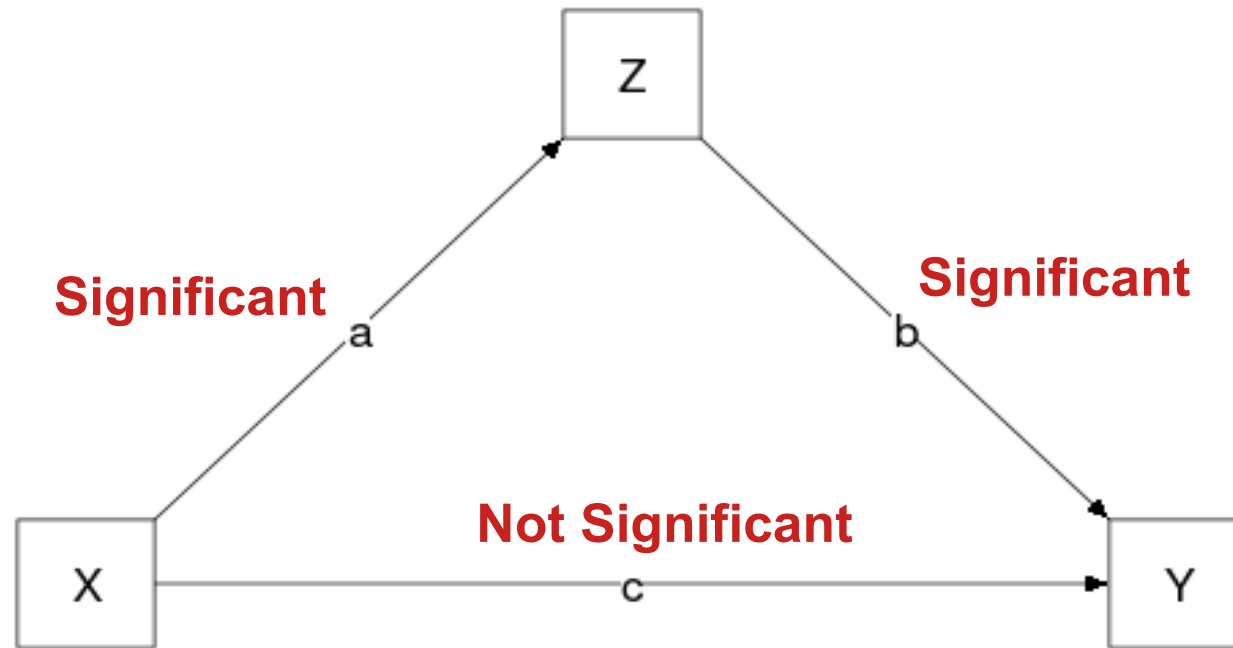


- If relationship between X and Y is mediated by Z, it indicates indirect causal effect of X to Y ( $X \rightarrow Z \rightarrow Y$ ).
- Total causal effect is the sum of direct and indirect effects of X and Z on Y Kline (2016)

# Mediation

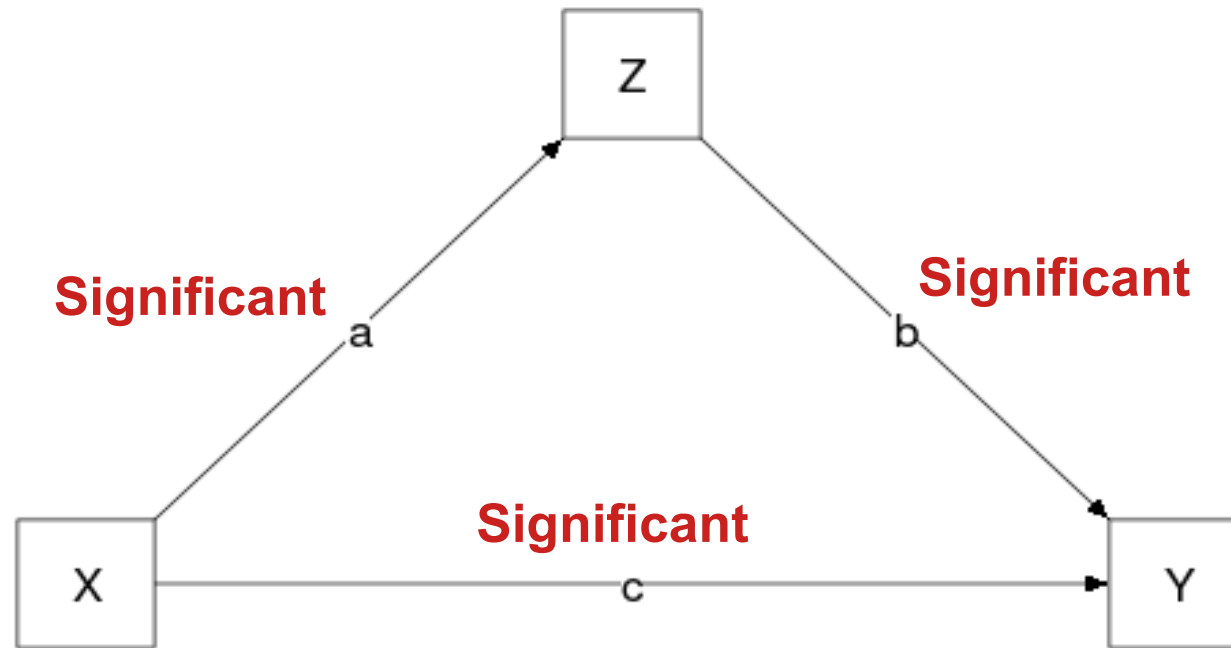
- Mediation effect can be classified into three outcomes (Awang, 2012):
  1. Complete mediation
  2. Partial mediation
  3. No mediation

# Complete Mediation



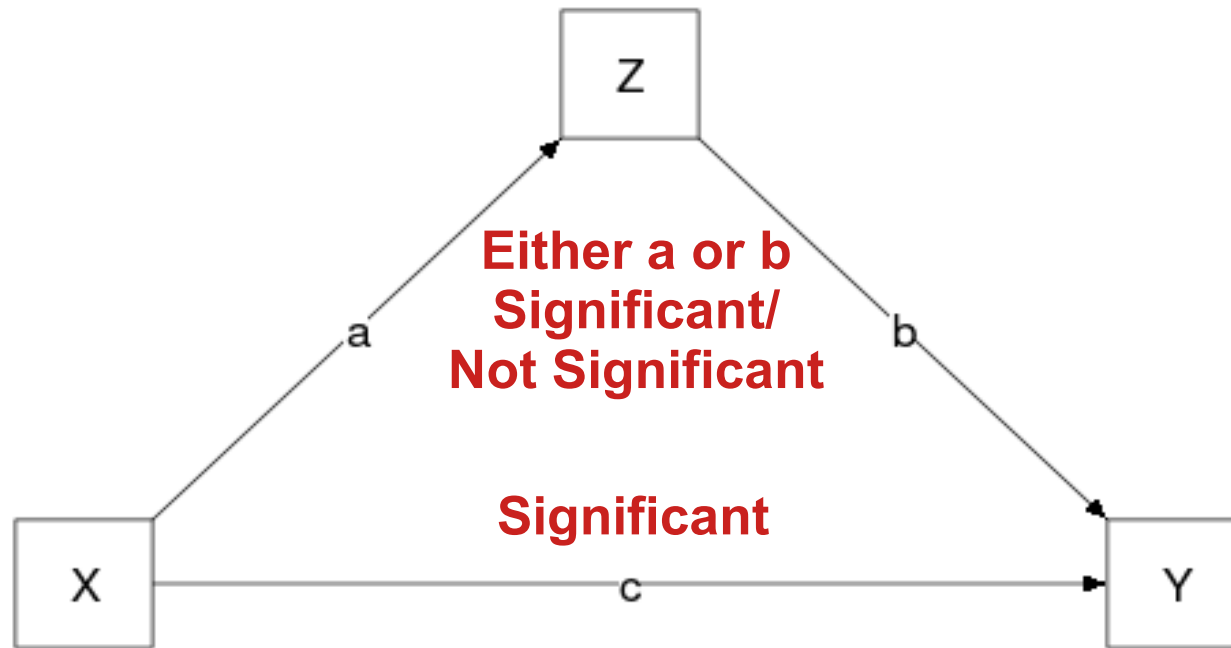
- Z completely mediates the effect of X on Y
- X indirectly causes Y

# Partial Mediation



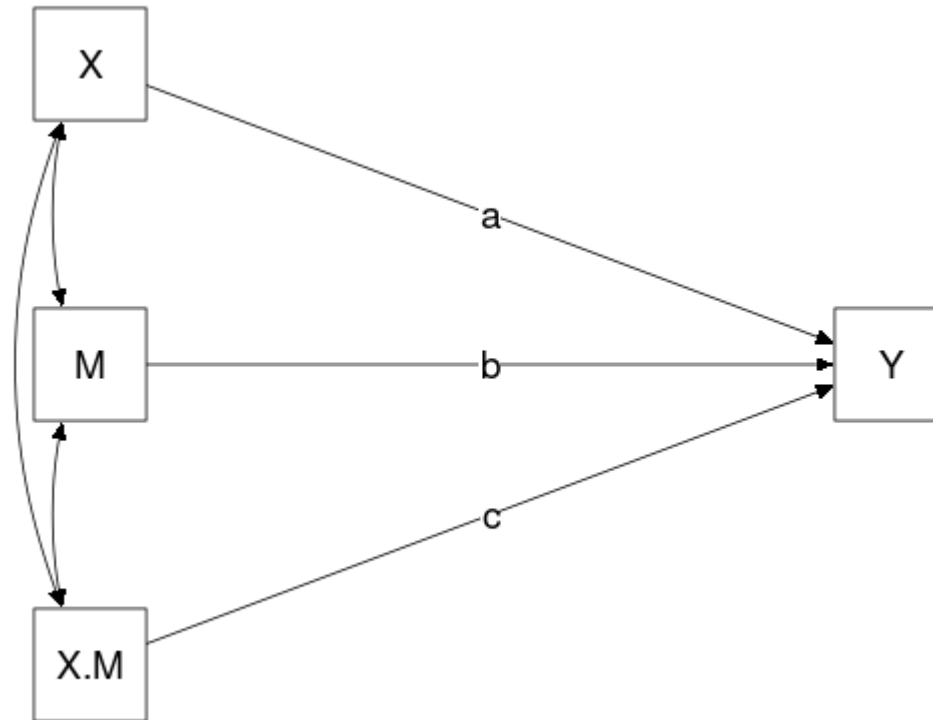
- Z partially mediates the effect of X on Y
- X directly and indirectly causes Y
- $c \text{ (with Z)} < c \text{ (without Z)}$  (Hair et al., 2010)

# No Mediation



- Z does not mediate the effect of X on Y
- X directly causes Y

# Moderation



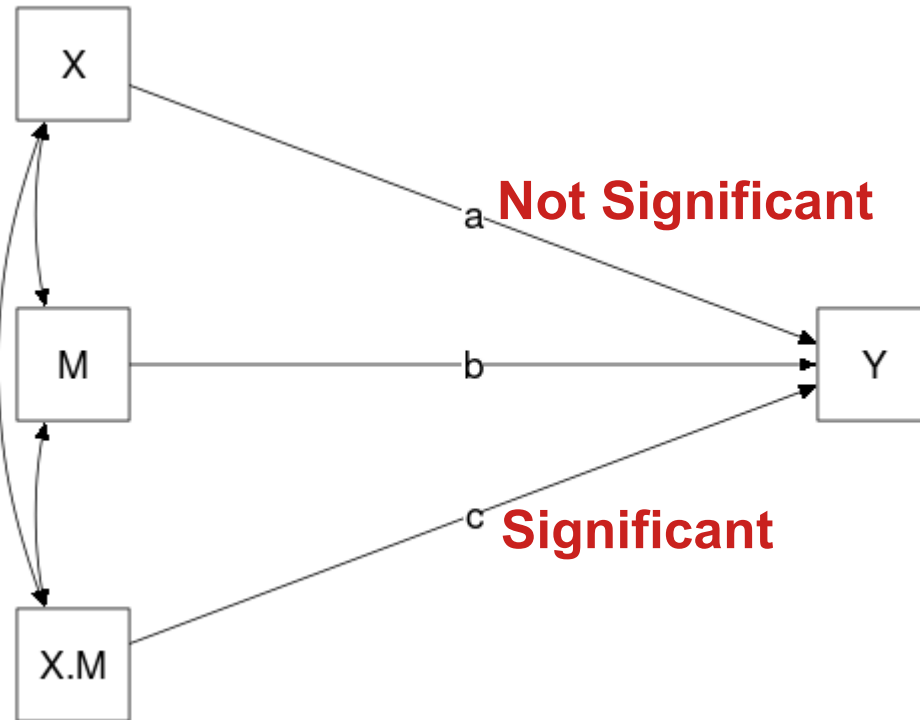
- A moderator M is a third variable that changes the relationship between two related variables X and Y (Hair et al., 2010)
- M moderates the causal effect that an IV has on a DV (Awang, 2012)
- Also commonly known as interaction



# Moderation

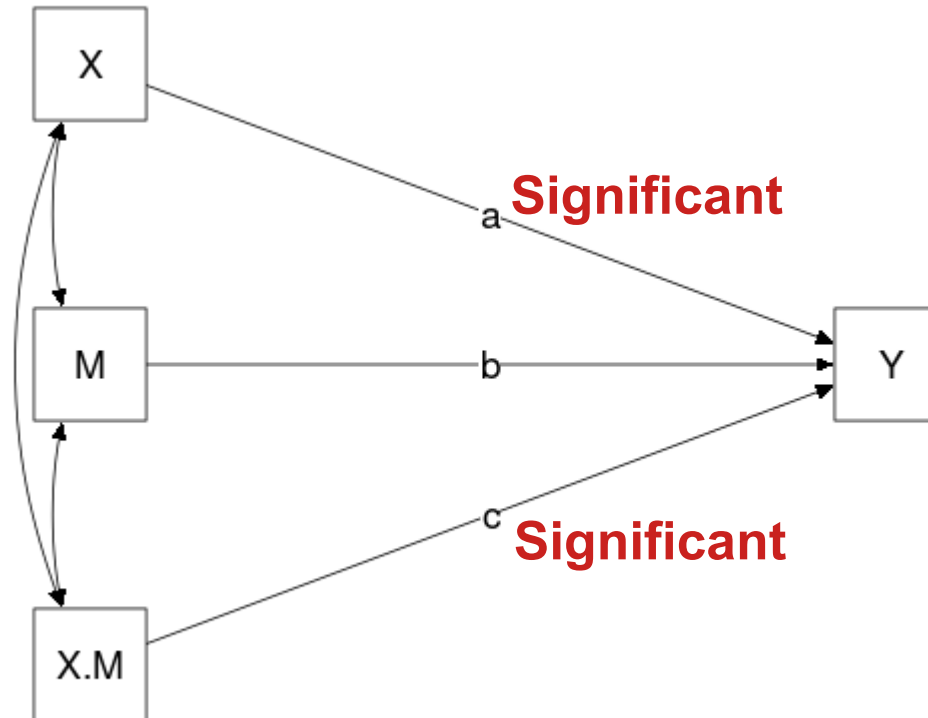
- Moderation effect can be classified into three outcomes (Awang, 2012):
  1. Complete moderation
  2. Partial moderation
  3. No moderation

# Complete Moderation



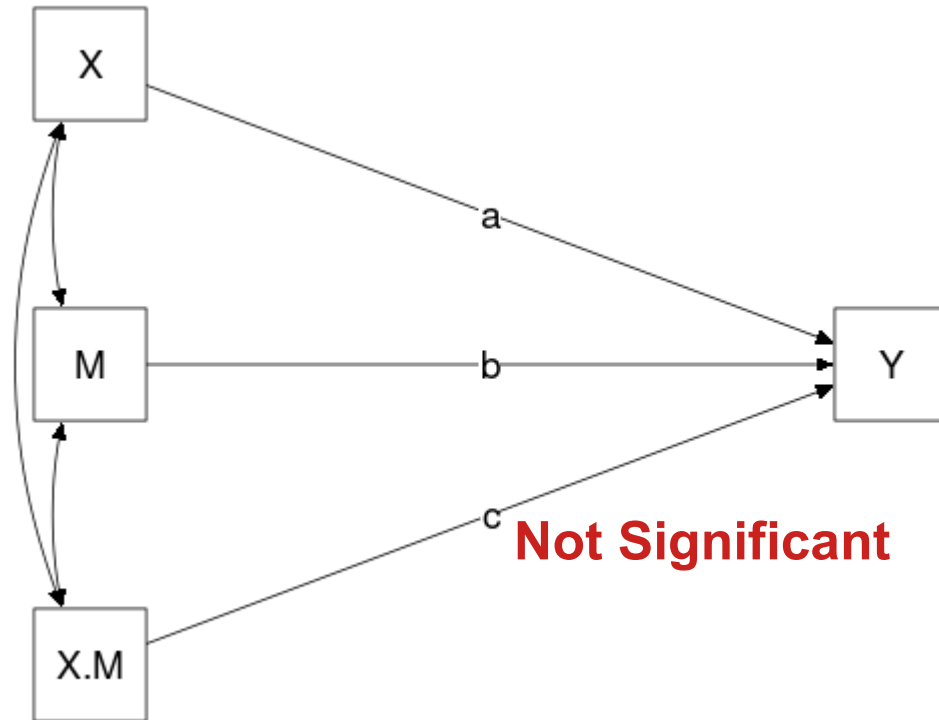
- M completely moderates the effect of X on Y
- M moderates the causal effect that an IV has on a DV (Awang, 2012)
- Also commonly known as interaction

# Partial Moderation



- M partially moderates the effect of X on Y
- $a \text{ (with } X*M) < a \text{ (without } X*M)$

# No Moderation



- M does not moderate the effect of X on Y
- $a \text{ (with } X*M) = a \text{ (without } X*M)$

# Moderation

- Moderation effect is typically applied to observed variables only
- For latent variables, it is complicated and requires:
  - multi-group CFA for categorical moderator (Hair et al. 2010; Awang, 2012)
  - adding a latent variable consisting of interaction items (e.g. for items in latent  $X^*M$ , items in latent  $X \times$  items in latent  $M$ ) in the model for numerical moderator (Hair et al., 2010)

# References

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